

No-Go Theorem for Critical Phenomena in Large- N_c QCD

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(Dated: March 19, 2012)

We derive some rigorous results on the chiral phase transition in QCD and QCD-like theories with a large number of colors, N_c , based on the QCD inequalities and the large- N_c orbifold equivalence. We show that critical phenomena and associated soft modes are forbidden in flavor-symmetric QCD at finite temperature T and finite but not so large quark chemical potential μ for any nonzero quark mass. In particular, the critical point in QCD at a finite baryon chemical potential $\mu_B = N_c\mu$ is ruled out, if the coordinate (T, μ) is outside the pion condensed phase in the corresponding phase diagram of QCD at a finite isospin chemical potential $\mu_I = 2\mu$.

PACS numbers: 11.10.Wx, 11.15.Pg, 12.38.Lg

Introduction.—The phase structure of quantum chromodynamics (QCD) at finite temperature T and finite baryon chemical potential μ_B is a longstanding problem despite its phenomenological importance, including heavy ion collisions and cosmology. Although it has been established from the first-principles lattice QCD simulations that the thermal chiral transition at $\mu_B = 0$ is a smooth crossover in real QCD [1], the fate of the chiral transition at nonzero μ_B has not been fully understood. In particular, not only the location, but even the existence of the QCD critical point(s) (see [2] for a review) has not yet been settled. This is mainly because the Monte Carlo method is not available at nonzero μ_B due to the sign problem.

One might hope that the $1/N_c$ expansion provides some new insights to this question, where the number of colors N_c is taken to infinity with keeping the 't Hooft coupling $\lambda = g^2 N_c$ fixed (the 't Hooft limit) [3]. This limit has proven successful for understanding of a number of aspects of hadrons in the QCD vacuum [4], and has also been widely applied to QCD at finite T and finite baryon chemical potential $\mu_B = N_c\mu$. The deconfinement temperature T_d is independent of μ (when $\mu \sim N_c^0$) in this limit since the gauge dynamics with $\sim N_c^2$ degrees of freedom is insensitive to the quark dynamics with $\sim N_c^1$ degrees of freedom [5]. However, the fate of the chiral phase transition is still an unanswered question. We only know that the critical temperature T_c of the chiral transition must satisfy $T_c \geq T_d$ because the chiral condensate $\sim N_c^1$ cannot be changed by non-interacting mesons and glueballs with $\sim N_c^0$ degrees of freedom in the confined phase [6].

In this Letter, we derive some exact results on the chiral phase transition in the large- N_c QCD and QCD-like theories. The critical phenomena (especially the QCD critical point) and associated soft modes are forbidden in flavor-symmetric QCD at finite μ_B for any nonzero quark mass m , as long as the coordinate (T, μ) is outside the pion condensed phase [7] in the corresponding phase diagram of QCD at finite isospin chemical potential $\mu_I = 2\mu$

(for the phase diagram, see Fig. 1 below).

Our results still allow a possibility that the QCD critical point exists inside the pion condensed phase in the large- N_c limit. Actually, in various effective models, such as the random matrix model [13], Nambu–Jona-Lasinio (NJL) model [14], and PNJL model [15], the critical point has been observed inside this region, as is consistent with our no-go theorem. This can be ascribed to the fact that the mean-field approximation in the model calculations corresponds to the leading order of $1/N$ expansion, and no-go theorems can also be formulated within these models [16]. Since the sign problem is maximally severe inside the pion condensed phase as pointed out by model analyses [13–15], our no-go theorem might imply that the conventional reweighting techniques are difficult to access the QCD critical point on the lattice. Our rigorous results are also useful to judge whether holographic models of QCD [17] motivated by the gauge/gravity duality [18] capture the genuine QCD physics or not.

QCD inequalities.—We first recall the rigorous QCD inequalities [19] which are essential in the discussion of this Letter. We shall work in the Euclidean and flavor-symmetric QCD with N_f flavors, and consider the Dirac

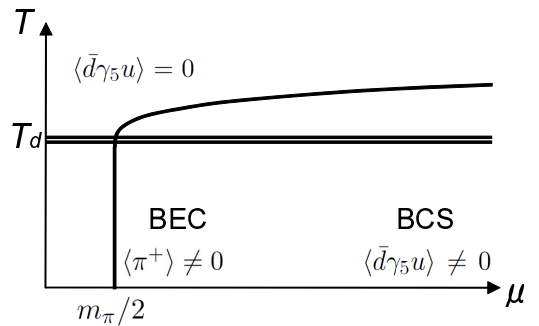


FIG. 1. Phase diagram of QCD with μ_I at large N_c . The single line denotes the pion condensed phase transition and the double line denotes deconfinement transition. (The chiral transition is not shown here.)

operator at $\mu_B = 0$, $\mathcal{D} = D + m$ with $D = \gamma_\mu(\partial_\mu + igA_\mu)$. The operator D satisfies the anti-Hermiticity and chiral symmetry, $D^\dagger = -D$ and $\gamma_5 D \gamma_5 = -D$. From these two properties, we have

$$\gamma_5 \mathcal{D} \gamma_5 = \mathcal{D}^\dagger, \quad (1)$$

and the positivity, $\det \mathcal{D} \geq 0$.

Let us take a generic flavor nonsinglet fermion bilinear $M_\Gamma = \bar{\psi} \Gamma \psi$ and consider a set of correlation functions,

$$\begin{aligned} C_\Gamma(x, y) &\equiv \langle M_\Gamma(x) M_\Gamma^\dagger(y) \rangle_{\psi, A} \\ &= -\langle \text{tr}[S_A(x, y) \Gamma S_A(y, x) \bar{\Gamma}] \rangle_A. \end{aligned} \quad (2)$$

Here, $S_A(x, y) \equiv \langle x | \mathcal{D}^{-1} | y \rangle$ is a propagator from y to x in a background gauge field A , the symbols $\langle \cdot \rangle_{\psi, A}$ and $\langle \cdot \rangle_A$ denote the full average and the average over the gauge field, respectively, and $\bar{\Gamma} \equiv \gamma_0 \Gamma^\dagger \gamma_0$. From (1) and the positivity of the measure, we have

$$\begin{aligned} C_\Gamma &= \langle \text{tr}[S_A(x, y) \Gamma i\gamma_5 S_A^\dagger(x, y) i\gamma_5 \bar{\Gamma}] \rangle_A \\ &\leq \langle \text{tr}[S_A(x, y) S_A^\dagger(x, y)] \rangle_A, \end{aligned} \quad (3)$$

where the Cauchy-Schwarz inequality is used. The inequality is saturated when $\Gamma = i\gamma_5 \tau_A$ with τ_A being the traceless flavor generators.

The asymptotic behavior of C_Γ at large distance $|x - y|$ can be written as

$$C_\Gamma \sim e^{-m_\Gamma |x - y|}, \quad (4)$$

with m_Γ being the mass of the lowest meson state in the channel Γ . Then the inequalities among correlators (3) lead to the inequalities among meson masses,

$$m_\Gamma \geq m_\pi, \quad (5)$$

where m_π is the mass of the pseudoscalar pion.

We note that the derivation of the QCD inequalities so far relies on the assumption that j_Γ is not flavor singlet. This condition is necessary, otherwise the flavor disconnected diagrams $\sim \langle \text{tr}[\Gamma S_A(x, x)] \text{tr}[\bar{\Gamma} S_A(y, y)] \rangle_A$, where $\bar{\psi}\psi$ turns into a gluonic intermediate state, also contribute and the above argument breaks down. Phenomenologically, the disconnected diagrams might be suppressed compared with the connected diagrams. A well-known example is the Okubo-Zweig-Iizuka (OZI) rule. Theoretically, it is the 't Hooft large- N_c limit [3] that cleanly justifies this statement [4]. The flavor disconnected diagrams are subleading compared with the connected diagrams in the $1/N_c$ expansion. Hence, at the leading order in the $1/N_c$ expansion, the QCD inequalities are also applicable to the flavor singlet channel. In particular, it follows that

$$m_\sigma \geq m_\pi. \quad (6)$$

Here, m_σ is the mass of the flavor singlet scalar σ which has the quantum number of the chiral condensate $\langle \bar{\psi}\psi \rangle$.

If one includes the subleading $1/N_c$ corrections that originate from the flavor disconnected diagrams, m_σ can be written as

$$m_\sigma = m_\pi + C + \mathcal{O}(N_c^{-1}), \quad (7)$$

with some $C \geq 0$. We here assume $m = \mathcal{O}(N_c^0)$, and thus $m_\pi = \mathcal{O}(N_c^0)$, since quark mass does not have any dependence of N_c in nature.

Chiral phase transition at $\mu_B = 0$.—Let us consider the thermal chiral phase transition in large- N_c QCD at $\mu_B = 0$ in the presence of nonzero quark mass m . In this case, a pion becomes massive due to the explicit breaking of chiral symmetry, $m_\pi > 0$. If there exists a second-order chiral transition at some critical temperature $T = T_c$ at $\mu_B = 0$, the screening mass in the σ -meson channel vanishes, $m_\sigma = 0$, or the correlation length $\xi = m_\sigma^{-1}$ is divergent [20].

However, the inequality (6), which is valid independent of T , leads to the finite bound $\sim N_c^0$ for m_σ ,

$$m_\sigma \geq m_\pi > 0. \quad (8)$$

This constraint clearly contradicts the fact that $m_\sigma = 0$ at $T = T_c$. Therefore, we arrive at the conclusion that the second-order chiral transition and associated soft modes are forbidden in the large- N_c QCD at $\mu_B = 0$ for any $m > 0$. This is our first no-go theorem. From this theorem, the large- N_c thermal chiral transition at $\mu_B = 0$ for $m > 0$ is either first order or crossover; we will discuss each possibility later.

Note that we used the large- N_c limit only to justify the suppression of flavor disconnected diagrams compared with flavor connected ones. This implies that, in real QCD with $N_c = 3$, the second-order chiral transition for $m > 0$ can happen, when the contribution of disconnected diagrams, the $\mathcal{O}(N_c^{-1})$ term in (7), cancels out that of connected diagrams, the remaining terms in (7).

Our no-go theorem agrees with the general argument on the order of the chiral phase transition based on the symmetries of QCD [21]. In three-color and three-flavor QCD, the thermal chiral transition at $\mu_B = 0$ becomes first order because of the $U(1)_A$ anomaly for small m , and becomes second order at some critical quark mass m_c . At large N_c , the anomaly effects related to disconnected diagrams are suppressed as $\mathcal{O}(N_c^{-1})$, and thus, $m_c = \mathcal{O}(N_c^{-1})$; the chiral transition is smeared into a crossover for any nonzero quark mass $\sim N_c^0$.

Chiral phase transition at $\mu_I \neq 0$.—The positivity of the theory, which enables us to use the QCD inequalities above, is essential to derive the no-go theorem for the chiral critical phenomena at $\mu_B = 0$. However, the Dirac operator at finite $\mu_B = N_c \mu$, $\mathcal{D}(\mu) = D + \mu \gamma_0 + m$ does no longer satisfy (1), and the positivity is lost; this is the origin of the notorious sign problem in QCD.

On the other hand, there are class of QCD-like theories that have the positivity at finite μ_B , such as QCD

with fermions in the adjoint representation [22], $\text{SO}(2N_c)$ [23, 24] and $\text{Sp}(2N_c)$ gauge theories [24]. Also QCD at finite isospin chemical potential μ_I maintains the positivity [25]. (We do not consider two-color QCD at finite μ_B , which also has the positivity [22, 26], because we cannot take the large- N_c limit in this theory.) We can apply our previous argument to these theories even at finite μ_B or finite μ_I , except the adjoint QCD. The reason why our argument fails in the adjoint QCD is that the flavor disconnected diagrams are not suppressed compared with the connected diagrams since “color” degrees of freedom of fermions $\sim N_c^2$ are comparable to those of gluons. Similarly, our argument is not applicable to QCD with fundamental quarks for fixed N_f/N_c and $N_c \rightarrow \infty$ [27] and QCD with two-index antisymmetric quarks for fixed N_f and $N_c \rightarrow \infty$ [28], the latter of which is used for the orientifold equivalence with adjoint QCD [29].

Let us take QCD at finite μ_I with two degenerate flavors as an example, which will be utilized later to generalize the no-go theorem to QCD at finite μ_B . The same argument is applicable to other theories, $\text{SO}(2N_c)$ gauge theory with any number of flavors and $\text{Sp}(2N_c)$ gauge theory with even number of flavors [30].

The Dirac operator in QCD at finite μ_I , $\mathcal{D}(\mu_I) = D + \mu_I \gamma_0 \tau_3 / 2 + m$ satisfies the relation [25]

$$\tau_1 \gamma_5 \mathcal{D} \gamma_5 \tau_1 = \mathcal{D}^\dagger, \quad (9)$$

for the degenerate quark mass m , from which the positivity $\det \mathcal{D}(\mu_I) \geq 0$ follows. One can then derive the inequality at *any* μ_I [25]:

$$C_\Gamma \leq \langle \text{tr}[S_A(x, y) S_A^\dagger(x, y)] \rangle_A, \quad (10)$$

where the inequality is saturated when $\Gamma = i\gamma_5 \tau_{1,2}$. This leads to the inequalities between meson masses in different channels, $m_\Gamma \geq m_{\pi_\pm}$. In the large N_c limit, QCD inequalities are applicable to the flavor singlet channel, and we have

$$m_\sigma \geq m_{\pi_\pm}. \quad (11)$$

Repeating a similar argument to QCD at $\mu_B = 0$, the second-order chiral transition is not allowed in this theory at finite μ_I where $m_{\pi_+} > 0$ or $m_{\pi_-} > 0$.

In order to consider the applicable region of the above no-go theorem, let us turn to the phase diagram of QCD at finite μ_I in the large- N_c limit shown in Fig. 1. (See [25] for the phase diagram with $N_c = 3$.) In this case, π_+ meson charged under isospin symmetry exhibits the Bose-Einstein condensation (BEC) $\langle \pi_+ \rangle \neq 0$ at low density, $\mu > m_\pi/2$ at $T = 0$ where the excitation energy $m_\pi - 2\mu$ becomes negative. On the other hand, at high density, the attractive interaction between quarks near the Fermi surface leads to the Bardeen-Cooper-Schrieffer (BCS) pairing of diquark with the quantum number $\langle \bar{d}\gamma_5 u \rangle$, as found from the weak-coupling calculations [25]. Because both condensates have the same quantum numbers and

break the same symmetry $U(1)_{L+R}$ down to \mathbb{Z}_2 , there should be no phase transition between the two regimes, similarly to the BEC-BCS crossover studied in nonrelativistic Fermi gases [31].

In Fig. 1, the deconfinement temperature T_d is independent of μ , as explained in the introduction. Also the critical chemical potential $\mu_c = m_\pi/2$ for the pion condensation is independent of T in the confined phase because of the large- N_c volume independence [32] or from the argument similar to the chiral condensate [6].

From the phase diagram of QCD at finite μ_I , $m_{\pi_\pm} > 0$ ($m_{\pi_+} = 0$ and $m_{\pi_-} > 0$) outside (inside) the pion condensed phase. Therefore, from the QCD inequalities (11), massless σ and the second-order chiral transition are prohibited in QCD at any μ_I for any $m > 0$.

Repeating the same argument in $\text{SO}(2N_c)$ and $\text{Sp}(2N_c)$ gauge theories at finite μ_B , the second-order chiral transition is forbidden in these theories. The locations of phase boundaries of the chiral transition, deconfinement transition, and BEC-BCS crossover region completely coincide with those of QCD at finite μ_I , as shown in [24].

Chiral phase transition at $\mu_B \neq 0$.—Now we are ready to generalize the no-go theorem to QCD at finite μ_B , using the results of QCD at finite μ_I . Based on the large- N_c orbifold equivalence [33], it was recently shown that a class of observables in QCD at finite μ_B , including the chiral condensate, exactly coincide those of QCD at finite μ_I outside the pion condensed phase. This can also be understood from the following argument: At the leading order of $1/N_c$, the contributions of up (u) and down (d) quarks to the chiral condensate are *decoupled* from each other, and hence, the chiral condensate does not distinguish the sign of the chemical potential for the d quark due to the charge conjugation symmetry. As a consequence, chiral condensate at finite μ_B and that at finite μ_I coincide. However, this argument fails if the pion condensation occurs in QCD at finite μ_I where u and d quarks are *coupled* in the ground state.

Since we already know from the argument above that chiral critical phenomena are forbidden in the large- N_c QCD at finite μ_I for $m > 0$, the same must be true in QCD at finite μ_B outside the pion condensed phase in the corresponding phase diagram of QCD at finite μ_I . Similarly one can obtain the no-go theorem in QCD at finite μ_B from that of $\text{SO}(2N_c)$ [or $\text{Sp}(2N_c)$] gauge theory at finite μ_B by using the large- N_c orbifold equivalence [23, 24] between these theories outside the diquark condensed phase. In particular, the equivalence with $\text{SO}(2N_c)$ gauge theory at finite μ_B leads to the stronger no-go theorem that, chiral critical phenomena are not allowed in massive and flavor-symmetric QCD with *any* number of flavors at finite μ_B , if the coordinate (T, μ) is outside the diquark condensed phase in the corresponding phase diagram of $\text{SO}(2N_c)$ gauge theory.

Chiral phase transition in the chiral limit.—Let us turn to the chiral transition in the chiral limit $m = 0$. We first

recall that chiral transition is either first or second order at $m = 0$. When $T_c > T_d$, we can utilize our no-go theorem for $m > 0$ to constrain the chiral transition at $m = 0$; a first-order chiral transition at $m = 0$ is prohibited by the no-go theorem because, with increasing m , it would eventually become second order at some critical $m = m_c$, while a second-order chiral transition at $m = 0$ is allowed because it would be smeared into a crossover for any nonzero $m = \mathcal{O}(N_c^0)$ [34]. When $T_c = T_d$, however, an interplay between chiral symmetry breaking and deconfinement always makes the chiral transition first order for any m [16] and is not constrained by the no-go theorem.

Therefore, we arrive at two possible scenarios for the chiral transition at $m = 0$: (a) the chiral transition is second order when $T_c > T_d$, or (b) the chiral transition is first order when $T_c = T_d$. Which scenario is realized in the large- N_c QCD cannot be determined from our arguments alone, and should be studied from other constraints or in the numerical lattice QCD simulations.

We thank A. Cherman, M. Hanada, K. Hashimoto, H. Iida, D. B. Kaplan, A. Karch, T. Morita, and D. T. Son for useful discussions, and T. Brauner and R. D. Pisarski for comments. N.Y. thanks the hospitality of Mathematical Physics Group at RIKEN Nishina Center where this work was initiated. Y.H. is supported by a Grant-in-Aid for Scientific Research by the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan (No.23340067). N.Y. is supported by JSPS Postdoctoral Fellowships for Research Abroad.

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